

CSULB Math Day at the Beach 2004, Individual Round, Multiple Choice

6. What is the value of the product $(\log_2 3) \cdot (\log_3 5) \cdot (\log_5 8)$?

- (a) 2 (b) 3 (c) 4 (d) 5 (e) 6

7. Which of the numbers below is a solution to the following equation?

$$\sqrt{4\Box\Box} + \sqrt{4\Box\Box} = x$$

- (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $\sqrt{\frac{15\Box\Box\sqrt{33}}{2}\Box}$ (d) $\sqrt{8\Box\Box\sqrt{2}}$ (e) 4

8. ABC is a triangle, and D is a point on segment BC . If $AB = 4$, $BD = 3$, $CD = 8$, and the area of triangle ABD is 5, what is the area of triangle ABC ?

- (a) 16 (b) 22 (c) $\frac{11\sqrt{7}}{2}$ (d) $\frac{40}{3}$ (e) $\frac{55}{3}$

9. Start with a square of area 5. Make a second square by joining together the midpoints of each of the sides of the first square. Then make a third square by joining together the midpoints of the sides of the second square. Continue this process indefinitely. What is the sum of the areas of all of these squares?

- (a) 0 (b) 5 (c) $\frac{10\Box\Box\sqrt{2}}{2}$ (d) 10 (e) infinite

10. For each real number a , we define $\Box a \Box$ to be the greatest integer which is less than or equal to a . For instance, $\Box 3.7 \Box = 3$ and $\Box 4 \Box = 4$. If x and y are two real numbers such that $\Box \sqrt{x} \Box = 9$ and $\Box \sqrt{y} \Box = 12$, then what is the largest possible value of $\Box x + y \Box$?

- (a) 225 (b) 242 (c) 256 (d) 268 (e) 269

11. Suppose f and g are invertible functions and suppose that $f(1) = 2$, $f(2) = 3$, $f(3) = 1$, $f(4) = 4$ and $g(1) = 2$, $g(2) = 4$, $g(3) = 1$, $g(4) = 3$. What is $f^{-1}(g^{-1}(2))$?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 1/2

12. If the width of a certain rectangle is tripled and the length is increased by 8, the area is quadrupled. What is the length of the original rectangle?

- (a) 8 (b) 6 (c) $16/3$ (d) 16 (e) 24

13. If $(1 + i)^{100}$ is expanded and written in the form $a + bi$, where a and b , are real, then $a =$?

- (a) -2^{50} (b) $20^{50} - \frac{100!}{50!50!}$ (c) $\frac{100!}{(25!)250!}$ (d) 2^{99} (e) 0

14. What is the number of distinct real solutions of the equation

$$x^4 + 10x^2 + 25 = 100x^2 - 400x + 400 ?$$

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

15. On a certain test, the average score for the women in the class is 83. The average score for the men in the class is 71. If the average score for the whole class is 80, what percentage of students in the class are women?

- (a) $33 \frac{1}{3} \%$ (b) 50% (c) $66 \frac{2}{3} \%$ (d) 75% (e) 80%

16. Suppose $\sin(x + y) = 0.8$ and $\sin(x - y) = 0.6$. Then $\cos x \sin y = ?$

- (a) -0.1 (b) 0.1 (c) 0.2 (d) 0.7 (e) 1

17. Let n equal the greatest integer that is less than or equal to $\sqrt{111,111,111,111}$. What is the remainder when n is divided by 7?

- (a) 0 (b) 1 (c) 3 (d) 4 (e) 6

18. Let n be the number such that $1 - 2 + 3 - 4 + \dots - (n - 1) + n = 2004$. What is the remainder when 4^{2004} is divided by n ?

- (a) 1 (b) 2 (c) 4 (d) $n - 2$ (e) $n - 1$

19. Suppose n is some 3-digit number. Let $m = (237n)^2$. Then m is a 9-digit number. Eight of the nine digits of m , in scrambled order, are 0, 0, 1, 4, 7, 8, 8, 9. What is the missing digit?

- (a) 1 (b) 2 (c) 4 (d) 6 (e) 8

20. Let $ABCD$ be the quadrilateral shown, where P is the intersection of the diagonals AC and BD . The area of $\triangle APB$ is 9 and the area of $\triangle CPD$ is 4. What is the minimum possible area of the quadrilateral $ABCD$?

- (a) 22 (b) 23 (c) 24
(d) 25 (e) 26

